Natural Sciences 102 Problem Set 4 Solutions

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Problem 1

We know that in every degree there are 60 minutes of arc and in every minute there are 60 seconds of arc, therefore

$$10 \text{ deg.} = 10 \text{ deg.} \times \frac{60 \text{ mins.}}{\text{deg.}} \times \frac{60 \text{ secs.}}{\text{min.}} = 36000 \text{ secs.} \quad (1)$$

Similarly, we know that π radians correspond to 180 degrees so

$$0.01 \text{ rad} \frac{180 \text{ deg.}}{\pi \text{ rad}} \times \frac{60 \text{ mins.}}{\text{deg.}} \times \frac{60 \text{ secs.}}{\text{min.}} = 2063 \text{ secs.} \quad (2)$$

Problem 2

a) A star has an stellar parallax of 0.1". What is its distance?

Answer: We know that using the parallax angle in arcseconds we can estimate the distance to the star in pc, using the relation

$$parallax - angle(arcseconds) = \frac{1}{distance(pc)} \quad (3)$$

so, if the angle is 0.1 arcseconds, then

$$distance(pc) = \frac{1}{0.1} = 10(pc) \quad (4)$$

the distance to the star is 10 pc.

b)A star is 30 pc distant. What is its annual stellar parallax?

Answer: We use the first formula given above, then

$$parallax - angle = \frac{1}{30} = 0.033 \quad (5)$$

then the annual parallax measured will be 0.033 arcseconds.

Problem 3

There is some confusion about which equation to use for parallax, so hopefully this will clear it up:

The simplest equation to use is

$$D = \frac{1}{P}, \quad (6)$$

where D is the distance in parsecs and P is the parallax angle in seconds of arc. Note that this equation **only** works if you use these units. That is, if you use P given in radians or degrees you will get the wrong answer.

Thus, for this example, we find

$$D = \frac{1}{P} = \frac{1}{0.02 \text{secondsofarc}} = 50 \text{parsecs.} \quad (7)$$

Angles given in radians are often useful because you can use the Law of Skinny Triangles (as given in class), which states

$$\tan P \approx \sin P \approx P, \quad (8)$$

whenever P is "small" and expressed in radians (not degrees or seconds of arc). Using this law, we can also find the distance to the star via

$$D = \frac{\text{Earth} - \text{Sundistance}}{\tan P} \approx \frac{\text{Earth} - \text{Sundistance}}{P(\text{inradians})}, \quad (9)$$

where now, you just have to be careful to express D and the Earth-Sun distance in the *same* units (not necessarily parsecs).

Problem 4

Angle in degrees	Angle in radians	Tangent of angle	Sine of angle
1^o	0.017453293	0.017455065	0.017452406
3^o	0.052359878	0.052407779	0.052335956
10^o	0.174532925	0.176326981	0.173648178
30^o	0.523598776	0.577350269	0.5
50^o	0.872664626	1.191753593	0.766044443

Problem 5

This problem involves a bit of manipulations. First of all note that the luminosity of the W star is 0.04 times the luminosity of the Sun, therefore $L_w = 0.04 L_\odot$, where the funny symbol $\odot$ is the standard symbol to refer to solar quantities. Furthermore we know that the distance of this star is $R_w = 1$ pc.$= 2 \cdot 10^5$ Au. We can then use the formula

$$m_\odot - m_w = -2.5 \log \left(\frac{I_\odot}{I_w} \right) \tag{10}$$

where the intensity I is related to the luminosity and the distance by the inverse square law

$$I = \frac{L}{4\pi R^2}. \tag{11}$$

Applying the above equation to the case at hand and expressing the distances in Au we have

$$\frac{I_\odot}{I_w} = \frac{L_\odot}{L_w} \frac{4\pi R_w^2}{4\pi R_\odot} = \frac{L_\odot}{0.04 L_\odot} \frac{(2 \cdot 10^5)^2}{1} = 10^{12}. \tag{12}$$

Going back to Eq. (10) and using the fact that $m_\odot = -26.8$ we then have

$$-26.8 - m_w = -2.5 \log(10^{12}) = -2.5 \cdot 12 \Rightarrow m_w = 3.2. \tag{13}$$

Finally finding the annual stellar parallax of W is straightforward since we know that it is 1 Pc away. So

$$D = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{D} \Rightarrow \alpha = 1 \text{ sec.} \tag{14}$$

Problem 6

a) The brightness of Arcturus and the light bulb are equal, therefore

$$\begin{aligned}
I_{arcturus} &= I_{bulb} \\
\frac{L_{arcturus}}{4\pi r_{arcturus}^2} &= \frac{L_{bulb}}{4\pi r_{bulb}^2} \\
\frac{r_{bulb}}{r_{arcturus}} &= \sqrt{\frac{L_{bulb}}{L_{arcturus}}} \\
\frac{r_{bulb}}{10\text{pc}} &= \sqrt{\frac{40\text{W}}{4 \times 10^{28}\text{W}}} \\
r_{bulb} &= 3.16 \times 10^{-13}\text{pc}
\end{aligned}$$

b) The distance to the light bulb expressed in cm is

$$\begin{aligned}
r_{bulb} &= 3.16 \times 10^{-13}\text{pc} \times \frac{4 \times 10^{18}\text{cm}}{1\text{pc}} \\
&= 1.26 \times 10^6\text{cm}
\end{aligned}$$